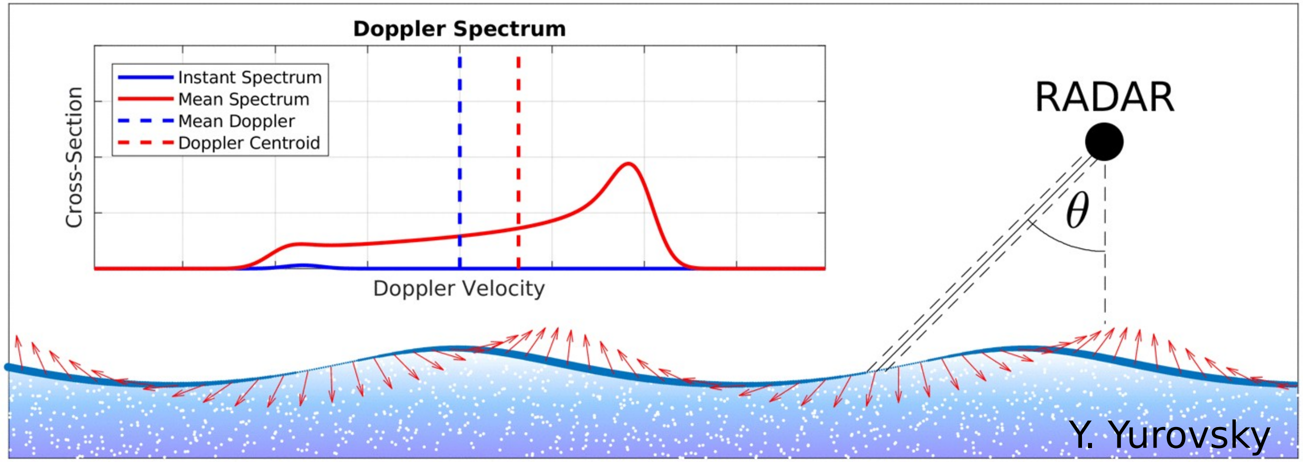
Theoretical developments



Nouguier, Chapron, Collard, Mouche, Rascle, Ardhuin, Wu (2018)
“Sea Surface Kinematics From Near-Nadir Radar Measurements”
IEEE TGRS

Wave Doppler (bias)

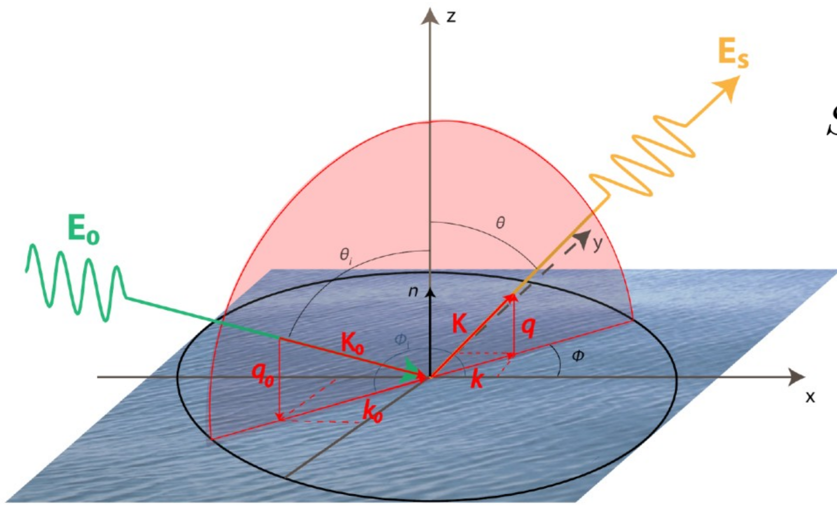


Figure 5: Diffraction (S. Guimbard)

$$S_{KA}(t) \propto \frac{\mathcal{K} e^{iK(R-ct)}}{Q_z (2\pi)^2} \int_A d\boldsymbol{r}\; e^{i\boldsymbol{Q_H}\cdot\boldsymbol{r}}\; e^{iQ_z \eta(\boldsymbol{r},t)}$$

$$\boldsymbol{Q_H} = \boldsymbol{k} - \boldsymbol{k_0}$$
$$Q_z = q + q_0$$

✓ **Sea-state Doppler frequency**

$$f_{\mathrm{GD}} = \frac{Q_z}{2\pi}\left(\mathrm{mss_{xt}}\,\frac{\partial_{\tan\theta}\,\psi^0}{\psi^0} + \frac{\mathrm{mss_{yt}}}{\tan\theta}\,\frac{\partial_{\varphi}\,\psi^0}{\psi^0}\right)$$

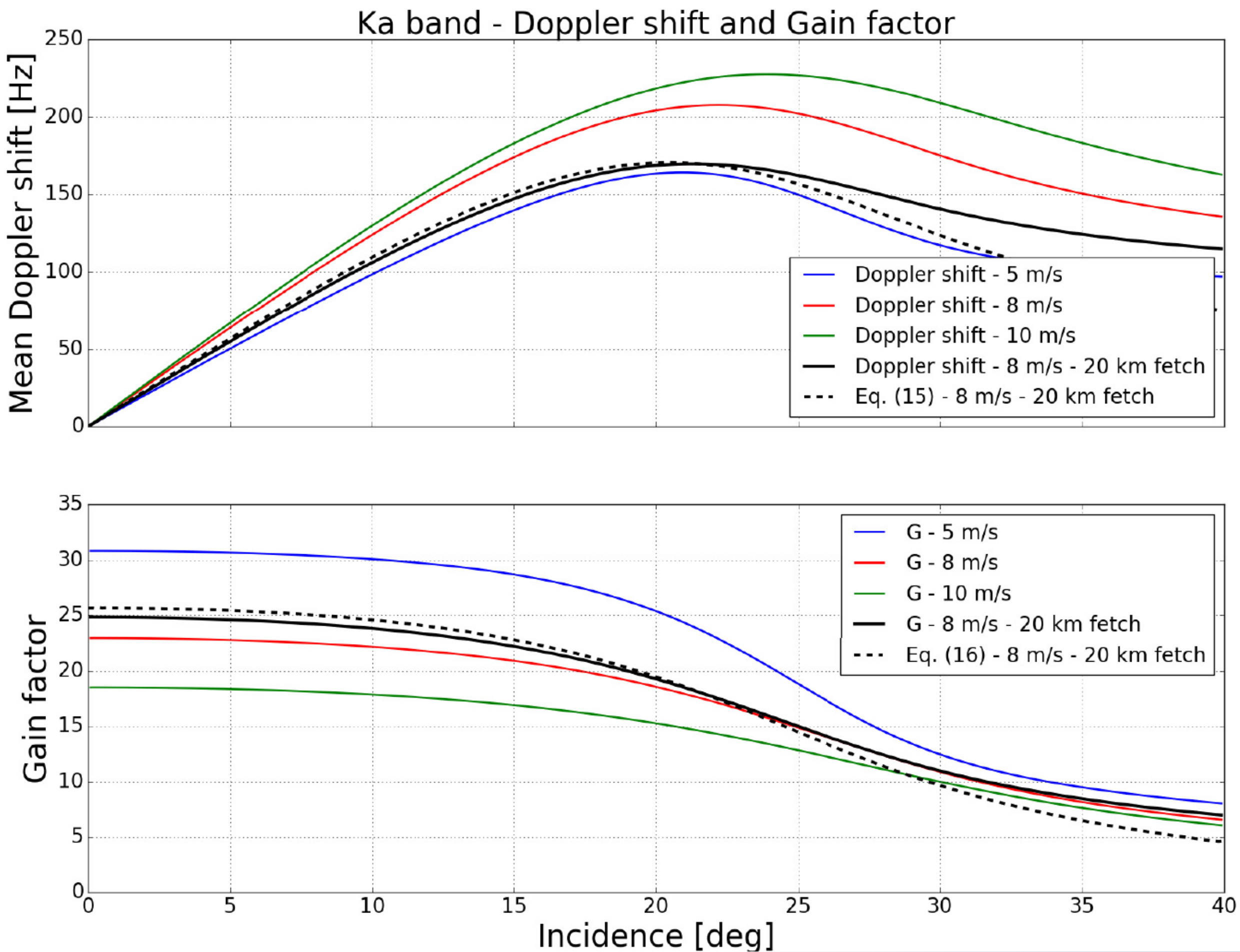
Mean range slope-velocity cross-correlation

NRCS incidence rate of variation

Mean azimuth slope-velocity cross-correlation

NRCS azimuthal rate of variation

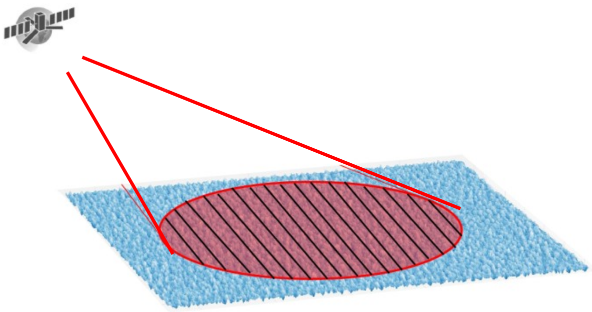
Wave Doppler (bias)



$$f_{\mathrm{GD}} = \frac{Q_z}{2\pi}\left(\mathrm{mss}_{\mathrm{xt}}\frac{\partial_{\tan\theta}\psi^0}{\psi^0} + \frac{\mathrm{mss}_{\mathrm{yt}}}{\tan\theta}\frac{\partial_{\varphi}\psi^0}{\psi^0}\right)$$

$$G = -\frac{1}{2}\frac{2\pi f_{\mathrm{GD}}}{Q_H \mathrm{mss}_{\mathrm{xt}}}$$

Range resolved measurements



$$\frac{\widetilde{\psi}^0(x)\widetilde{f}_{GD}(x)}{(2\pi)^{-1}\psi^0} = Q_z \frac{\partial_{\tan\theta}\psi^0}{\psi^0}\mathrm{mss}_{xt}$$

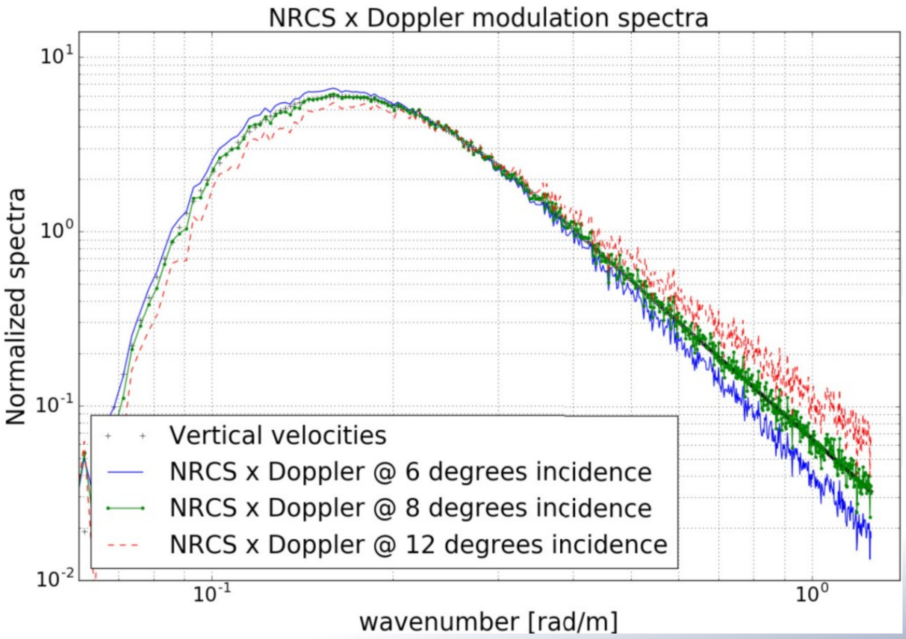
$$- Q_z\frac{\partial_{\tan\theta}\psi^0}{\psi^0}\overline{\partial_x\eta_r}(x)\overline{\partial_t\eta_r}(x)$$

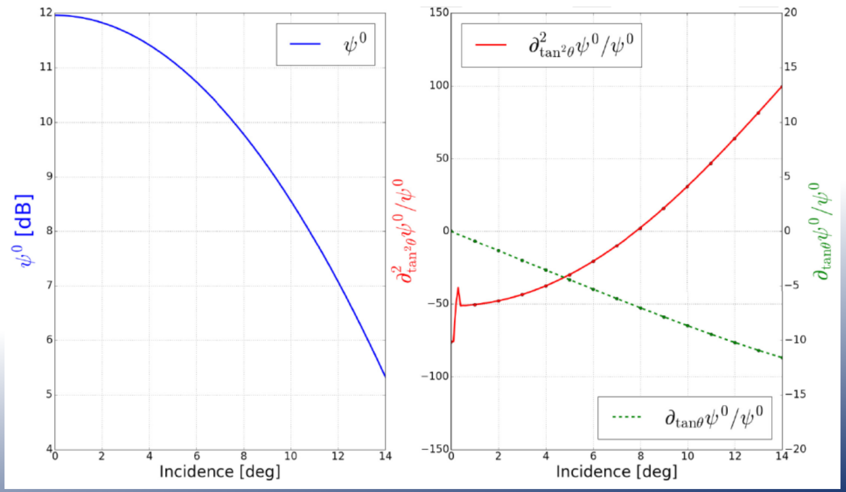
$$+ Q_z\overline{\partial_t\eta_r}(x) - Q_z \mathrm{mss}_{xt}\overline{\partial_x\eta_r}(x)\frac{\partial^2_{\tan^2\theta}\psi^0}{\psi^0}.$$

Mean wave Doppler

Slopes contribution

Resolved waves vertical velocities





A short focus on the equations

✓ **Sigma0 modulations (basic SWIM concept equation):**

$$\widetilde{\sigma}^0(x) = \sigma^0 \left(1 + \overline{\partial_x \eta_l}(x) \frac{\partial_{\tan\theta}\sigma^0}{\sigma^0}\right)$$

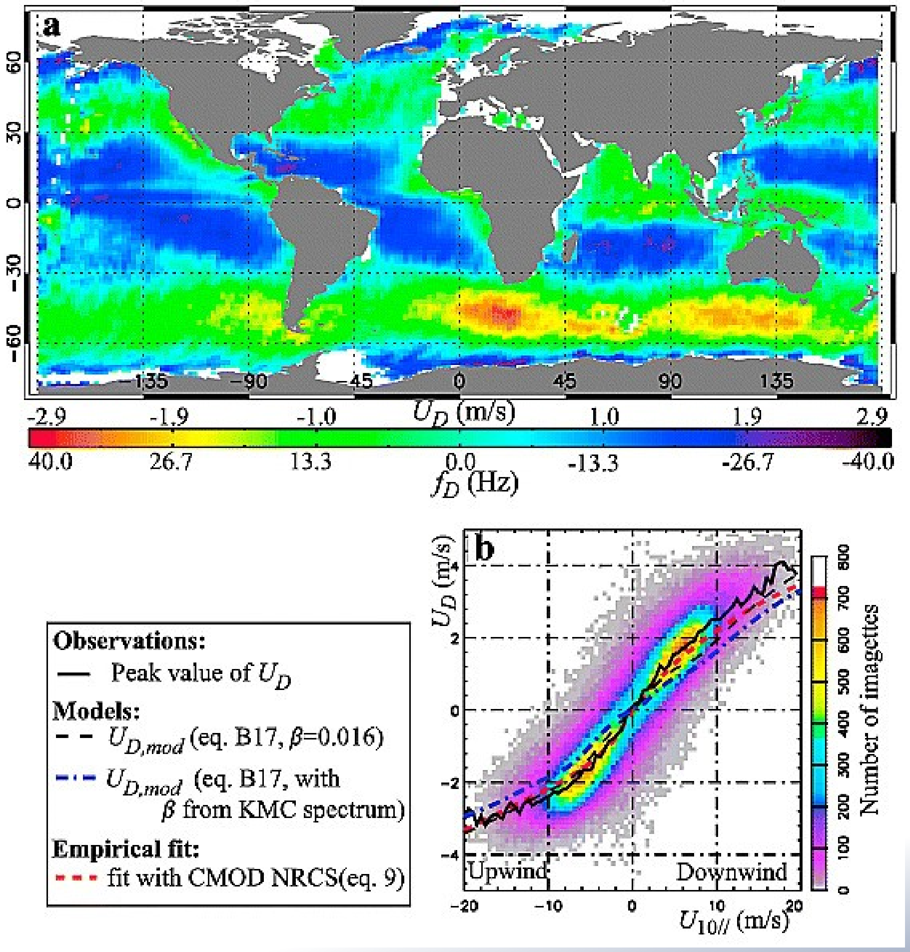
Range resolved
- wave slopes (red circles)
- wave velocity (green circles)

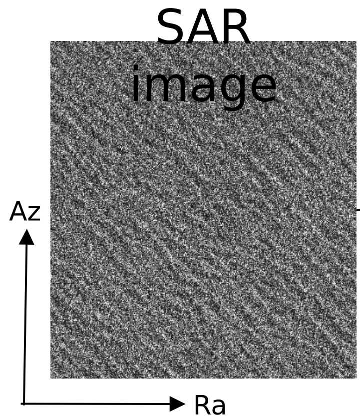
✓ **Doppler (velocity) modulations (basic SKIM equation):**

$$2\pi \widetilde{f_{GD}}(x) = Q_z \overline{\partial_t \eta_l}(x) + \frac{Q_z mss_{xt}}{\widetilde{\sigma}^0(x)} \left[\partial_{\tan\theta}\sigma^0 + \overline{\partial_x \eta_l}(x) \frac{\partial^2_{\tan^2\theta}\sigma^0}{\sigma^0}\right]$$

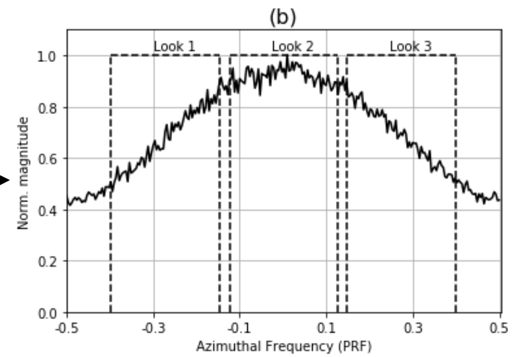
✓ **Weighted Doppler modulations :**

$$2\pi \frac{\widetilde{\sigma}^0(x)\widetilde{f_{GD}}(x)}{\sigma^0} = Q_z \overline{\partial_t \eta_l}(x) + Q_z mss_{xt} \overline{\partial_x \eta_l}(x) \frac{\partial^2_{\tan^2\theta}\sigma^0}{\sigma^0} + \ldots$$


a
U_D (m/s)
-2.9 -1.9 -1.0 1.0 1.9 2.9
40.0 26.7 13.3 0.0 -13.3 -26.7 -40.0
f_D (Hz)
b
Observations:
Peak value of U_D
Models:
$U_{D,mod}$ (eq. B17, β=0.016)
$U_{D,mod}$ (eq. B17, with β from KMC spectrum)
Empirical fit:
fit with CMOD NRCS(eq. 9)
U_D (m/s)
Upwind
Downwind
$U_{10//}$ (m/s)
Number of imagettes



FT in azimuth



Inverse FT

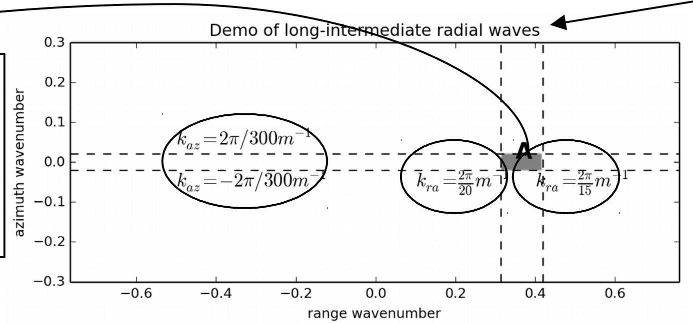
Look -1	$I_1(\vec{x}, t_1)$
Look -2	$I_2(\vec{x}, t_2)$
Look -3	$I_3(\vec{x}, t_3)$

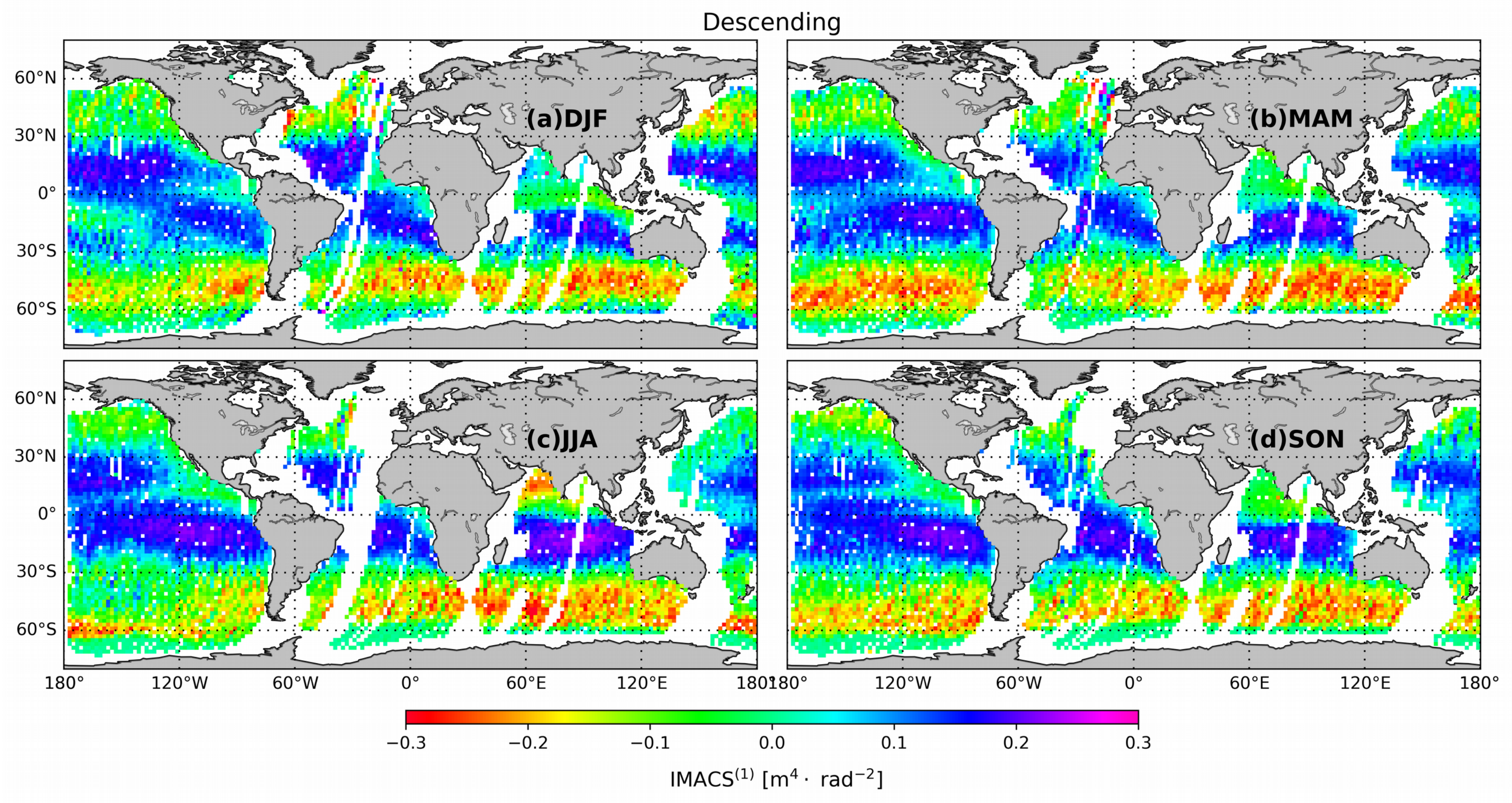
$$P_s^{(m,n)}(\vec{k}, \Delta t) = I_{(m)}(\vec{k}, t_m) \cdot I^*_{(n)}(\vec{k}, t_n), \quad m, n \in [1, 2, 3]$$

$$I_{(m)}(\vec{k}, t_m) = FT\{I_m(\vec{x}, t_m)\} \quad m \in [1, 2, 3]$$

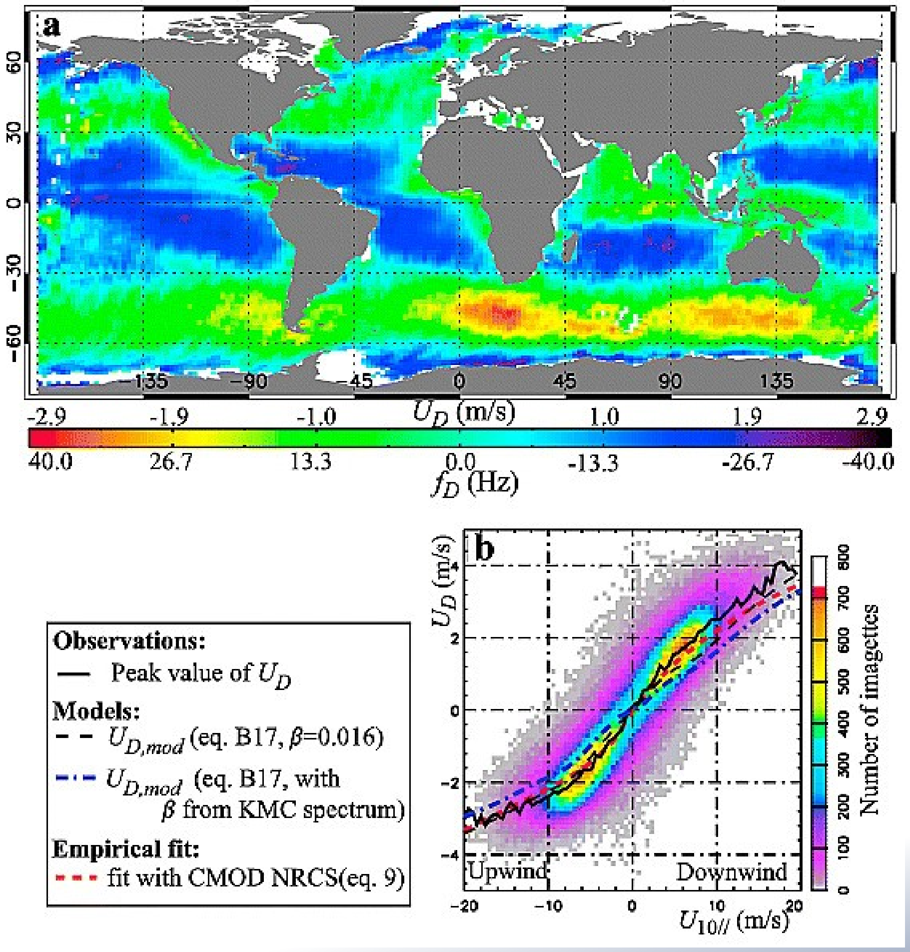
Δt is the separation time between two sublooks.

MeAn Cross-spectra (MACS) over range intermediate waves





Descending
(a)DJF
(b)MAM
(c)JJA
(d)SON
60°N
30°N
0°
30°S
60°S
180°
120°W
60°W
0°
60°E
120°E
180°
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
IMACS(1) [m4 · rad−2]

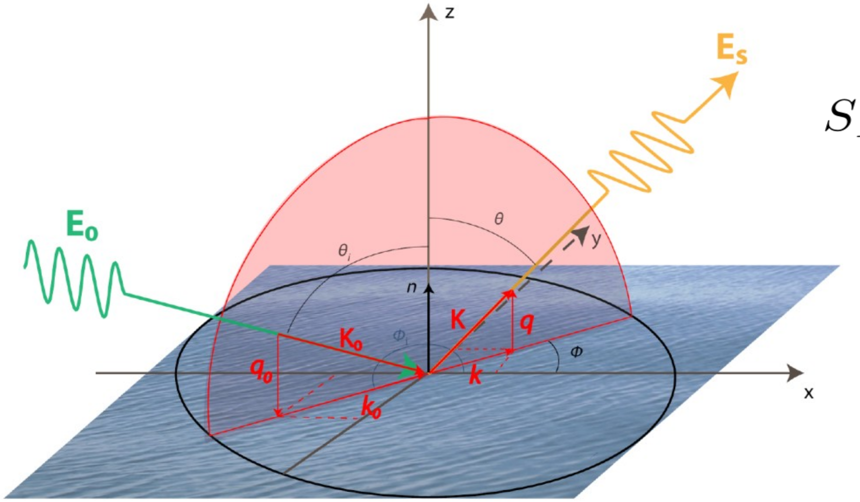

a
60
30
0
−30
−60
−135
−90
−45
0
45
90
135
-2.9
-1.9
-1.0
U_D (m/s)
1.0
1.9
2.9
40.0
26.7
13.3
0.0
-13.3
-26.7
-40.0
f_D (Hz)
Observations:
Peak value of U_D
Models:
$U_{D,mod}$ (eq. B17, β=0.016)
$U_{D,mod}$ (eq. B17, with β from KMC spectrum)
Empirical fit:
fit with CMOD NRCS(eq. 9)
b
U_D (m/s)
4
2
0
-2
-4
Upwind
Downwind
−20
−10
0
10
20
$U_{10//}$ (m/s)
0
100
200
300
400
500
600
700
800
Number of imagettes

Theoretical developments

for

Δk technique

Delta-k principle



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$$\boldsymbol{Q_H} = \boldsymbol{k} - \boldsymbol{k_0}$$

$$Q_z = q + q_0$$

$$\begin{cases} \Delta K & = & |\boldsymbol{K}^+| - |\boldsymbol{K}^-| \\ \boldsymbol{Q_H^{\pm}} & = & \boldsymbol{Q_H} \pm \Delta \boldsymbol{Q_H} \\ Q_z^{\pm} & = & Q_z \pm \Delta Q_z \end{cases}$$

✓ **Delta-k correlation function:**

$$C^{\Delta k}(\tau) = \left\langle \boldsymbol{E}(\boldsymbol{K}^+, t)\boldsymbol{E}^*(\boldsymbol{K}^-, t+\tau) \right\rangle$$

Delta-k results

$$C^{\Delta k}(\boldsymbol{Q}_H, Q_z, \tau, \nu) \propto \psi e^{2\Delta Q_z^2 \rho(\mathbf{0},0)} \int_{L_x} e^{-2i\Delta Q_H \chi} e^{2i\Delta Q_z \overline{\eta}(\chi,\nu)} e^{iQ_z \tau \partial_t \overline{\eta}(\chi,\nu)} d\chi \times \int_A e^{-i(Q_H - Q_z \partial_x \overline{\eta}(\chi,\nu))\xi_x} e^{(Q_z^2 - \Delta Q_z^2)(\rho(\mathbf{0},0) - \rho(\boldsymbol{\xi},\tau))} d\boldsymbol{\xi}$$

$$\tau = t - t'$$
$$\boldsymbol{\xi} = (\xi_x, \xi_y) = (x - x', y - y')$$
$$\nu = t + t'$$
$$\chi = x + x'$$

Mean range elevation profile

Mean range vertical velocities profile

Mean range slope profile

Surface correlation function

Delta-k results

$$C^{\Delta k}(\boldsymbol{Q}_H, Q_z, \tau, \nu) \propto \psi e^{2\Delta Q_z^2 \rho(\mathbf{0},0)} \int_{L_x} e^{-2i\Delta Q_H \chi} e^{2i\Delta Q_z \overline{\eta}(\chi,\nu)} e^{iQ_z \tau \partial_t \overline{\eta}(\chi,\nu)} d\chi \times$$

$$\int_A e^{-i(Q_H - Q_z \partial_x \overline{\eta}(\chi,\nu))\xi_x} e^{(Q_z^2 - \Delta Q_z^2)(\rho(\mathbf{0},0) - \rho(\boldsymbol{\xi},\tau))} d\boldsymbol{\xi}$$

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Mean range elevation profile
Mean range vertical velocities profile
Mean range slope profile
Surface correlation function

Temporal co-location of Δk Echos

$$\tau = 0$$

$$C^{\Delta k}(\nu) \simeq \psi e^{2\Delta Q_z^2 \rho(\mathbf{0},0)} \int_{L_x} e^{-2i\Delta Q_H \chi} (1 + 2i\Delta Q_z \overline{\eta}(\chi, \nu)) \tilde{\zeta}^0(\chi, \nu) d\chi$$

Delta-k results

$$C^{\Delta k}(\boldsymbol{Q}_H, Q_z, \tau, \nu) \quad \propto \quad \psi e^{2\Delta Q_z^2 \rho(\mathbf{0},0)} \int_{L_x} e^{-2i\Delta Q_H \chi} e^{2i\Delta Q_z \overline{\eta}(\chi,\nu)} e^{iQ_z \tau \partial_t \overline{\eta}(\chi,\nu)} d\chi \times$$
$$\int_A e^{-i(Q_H - Q_z \partial_x \overline{\eta}(\chi,\nu))\xi_x} e^{(Q_z^2 - \Delta Q_z^2)(\rho(\mathbf{0},0) - \rho(\boldsymbol{\xi},\tau))} d\boldsymbol{\xi}$$

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Temporal co-location of Δk Echos

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$$D(\omega) = \int e^{i\omega\nu} C^{\Delta k}(\nu) d\nu$$

HF-like Doppler spectrum which peaks at ΔQ_H wave frequency